Lezioni dottorali, Università degli Studi di Napoli "Parthenope"

Napoli, 21–27 October 2016

Nonlinearity, Tipping Points & Chaos in the Climate Sciences

Michael Ghil

Ecole Normale Supérieure, Paris, and
University of California, Los Angeles

Please visit these sites for more info.

https://dept.atmos.ucla.edu/tcd

http://www.environnement.ens.fr/

Lezioni dottorali, Università degli Studi di Napoli "Parthenope"

Lecture III, Napoli, 26 October 2016

Time Series Analysis and Applications to the Climate Sciences and to Other Fields

Michael Ghil

Ecole Normale Supérieure, Paris, and University of California, Los Angeles

Joint work with many people; most recently with
A. Groth & D. Kondrashov (UCLA)

ENS

Please visit these sites for more info.
https://dept.atmos.ucla.edu/tcd,, http://www.environnement.ens.fr/
and https://www.researchgate.net/profile/Michael_Ghil

Advanced Spectral Methods, Nonlinear Dynamics, the Geosciences and Econometrics

Motivation

- Time series in the geosciences and elsewhere have typically broad peaks on top of a continuous, "warm-colored" background ➔ ***Method***
- Connections to nonlinear dynamics ➔ ***Theory***
- Need for stringent statistical significance tests ➔ ***Toolkit*** (*)
- Applications to analysis and prediction ➔ ***Examples***

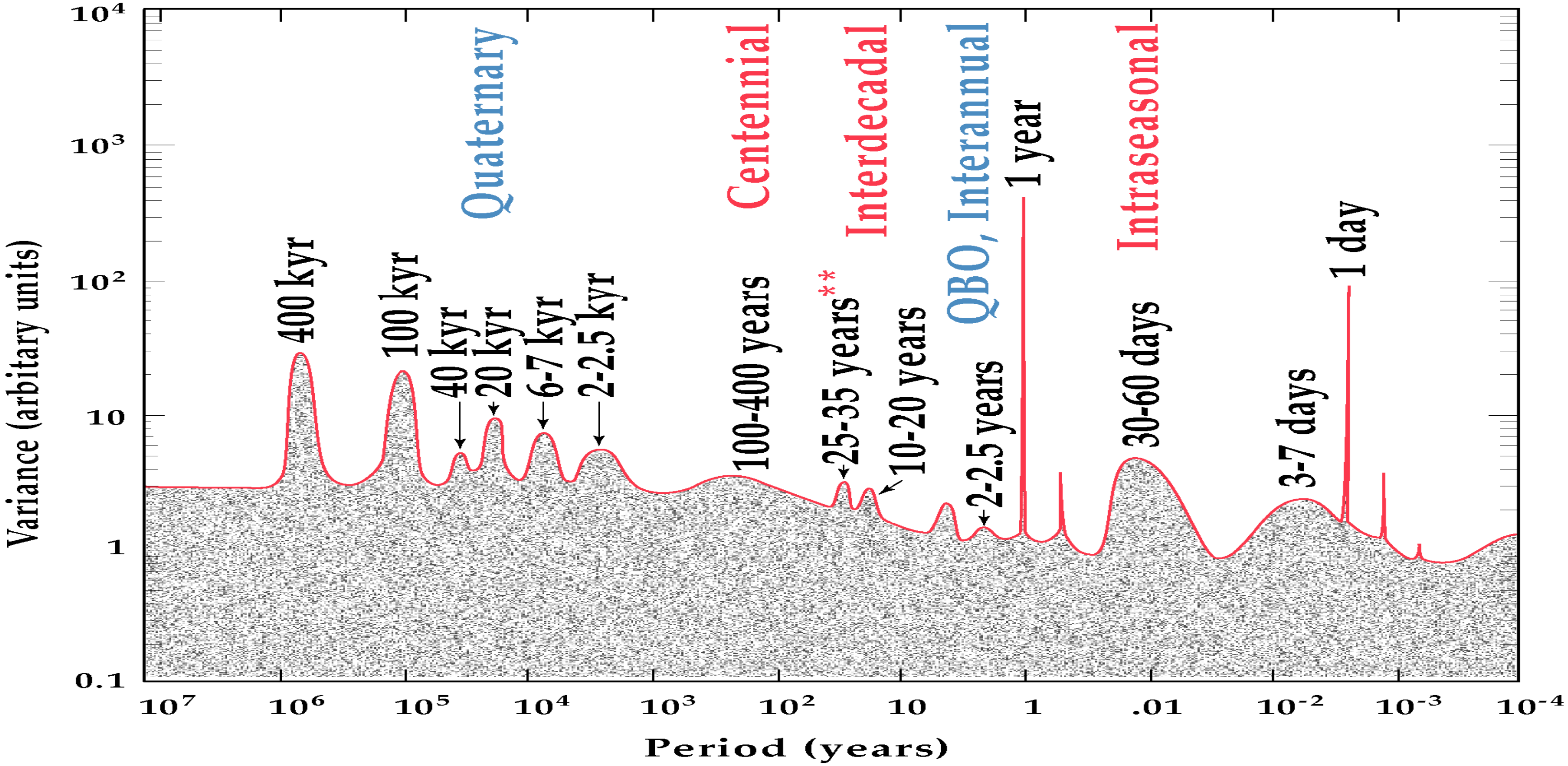
(*) *SSA-MTM Toolkit*, https://dept.atmos.ucla.edu/tcd/ssa-mtm-toolkit

Composite spectrum of climate variability

Standard treatement of frequency bands:

1. High frequencies – white noise (or "colored")

2. Low frequencies – slow evolution of parameters



From **Ghil (2001, *EGEC*)**, after **Mitchell* (1976)**

*** "No known source of deterministic internal variability"**

**** 27 years – Brier (1968, *Rev. Geophys.*)**

Outline

- **Motivation**: spectrum has trends, broad peaks, and continuous background + connections to nonlinear dynamics.
- **Methodology**
  - – univariate: singular-spectrum analysis (SSA)
  - – multivariate: M-SSA + varimax rotation
  - – statistical significance tests: MC-SSA & Procrustes rotation
  - – SSA-MTM Toolkit
  - – gap filling
- **Applications** to analysis and prediction
  - – Southern Oscillation Index (SOI)(*)
  - – Nile River floods
  - – GPS data sets in the geosciences
  - – macroeconomic indicators – US, EU & global
- **Concluding remarks and bibliography**

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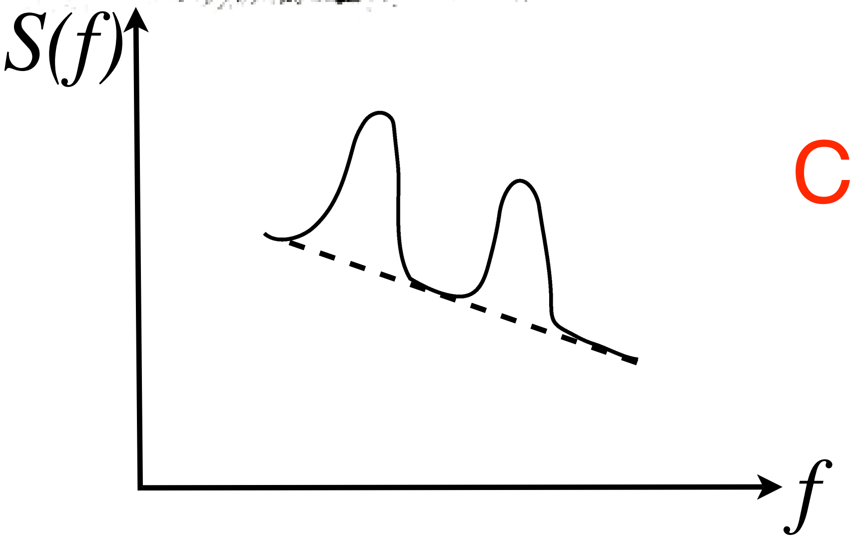
Spectral Density (Math)/Power Spectrum (Science & Engng.)



Continuous background
+ peaks

- **Wiener-Khinchin (Bochner) Theorem** **Blackman-Tukey Method**

$$R(s) = \lim_{L\to\infty} \frac{1}{2L} \int_{-L}^{L} x(t)x(t+s)dt$$

$$S(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(s)e^{-ifs}ds \equiv \hat{R}(s)$$

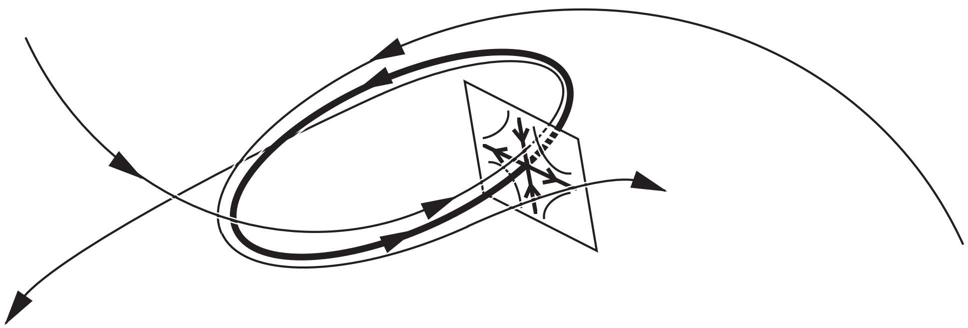
i.e., the **lag-autocorrelation function** & the **spectral density**

are **Fourier transforms** of each other.

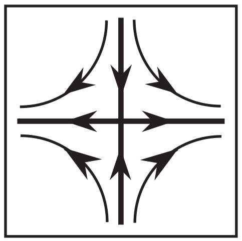
Power Law for Spectrum (cont'd)

- Hypothesis: "***Poles***" correspond to the least unstable periodic orbits

"***unstable limit cycles***"



"***Poincaré section***"



- Major clue to the physics

  that underlies the dynamics

- N.B. Limit cycle not necessarily elliptic, i.e. not

$$(x, y) = (a_f sin(ft), b_f cos(ft))$$

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Singular

Spectrum

Analysis

Singular Spectrum Analysis (SSA)

Spatial EOFs

x -- *space*

$$\phi(x,t) = \sum a_k(t) e_k(x)$$

$$C_\phi(x,y) = E\phi(x,\omega)\phi(y,\omega) = \frac{1}{T}\int_o^T \phi(x,t)\phi(y,t)\,dt$$

$$C_\phi e_k(x) = \lambda_k e_k(x)$$

Colebrook (1978); *Weare & Nasstrom* (1982);
Broomhead & King (1986: BK); *Fraedrich* (1986)

BK+VG: Analogy between Mañe-Takens embedding and the Wiener-Khinchin theorem

SSA

s -- *lag*

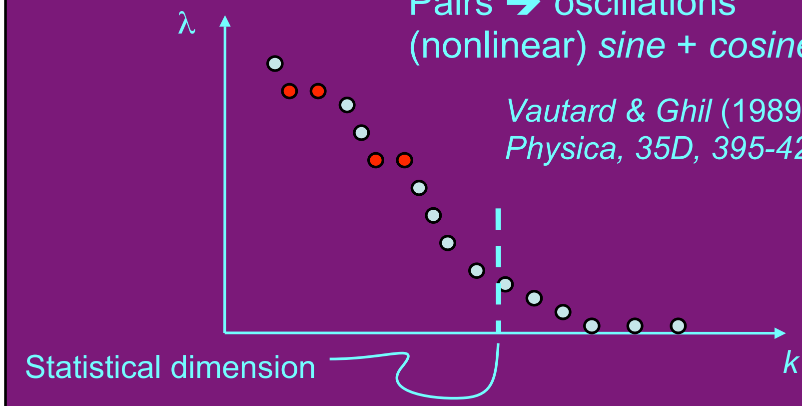
$$X(x+s) = \sum a_k(t) e_k(s)$$

$$C_X(s) = EX(t+s,\omega)\phi(s,\omega) = \frac{1}{T}\int_o^T X(t)X(t+s)\,dt$$

$$C_X e_k(s) = \lambda_k e_k(s)$$

Pairs ➔ oscillations
(nonlinear) *sine* + *cosine* pair

Vautard & Ghil (1989: VG)
Physica, 35D, 395-424



Power Spectra & Reconstruction

- **A. Transform pair**:

$$X(t+s) = \sum_{k=1}^{M} a_k(t) e_k(s), e_k(s) - EOF$$

The e_k's are **adaptive filters**,

$$a_k(t) = \sum_{s=1}^{M} X(t+s) e_k(s), a_k(t) - PC$$

the a_k's are **filtered time series**.

B. Power spectra

$$S_X(f) = \sum_{k=1}^{M} S_k(f); \quad S_k(f) = R_k(s); \quad R_k(s) \approx \frac{1}{T}\int_0^T a_k(t) a_k(t+s) dt$$

C. Partial reconstruction

$$X^K(t) = \frac{1}{M} \sum_{k \in K} \sum_{s=1}^{M} a_k(t-s) e_k(s);$$

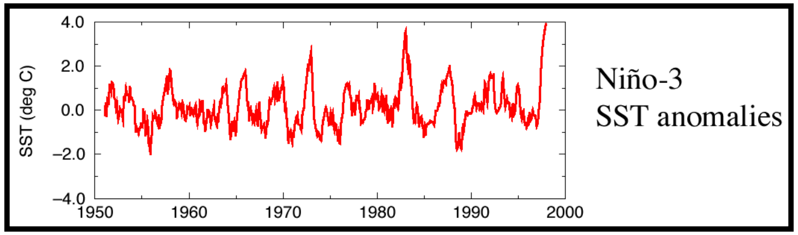
in particular: $K = \{1, 2, ..., S\}$ or $K = \{k\}$ or $K = \{l, l+1; \lambda_l \approx \lambda_{l+1}\}$

Singular Spectrum Analysis (SSA)

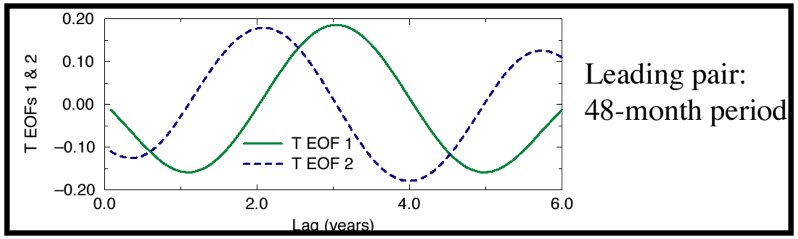
SSA decomposes (geophysical & other) time series into *Temporal EOFs* (T-EOFs) and *Temporal Principal Components* (T-PCs), based on the series' lag-covariance matrix

Selected parts of the series can be reconstructed, via *Reconstructed Components* (RCs)

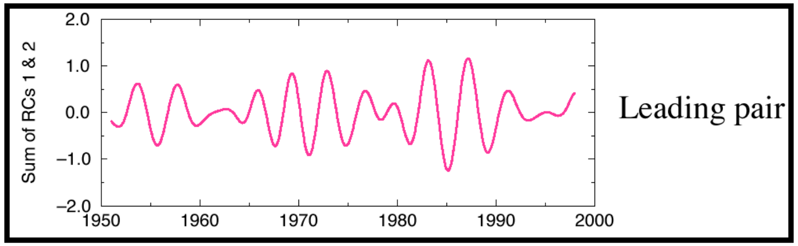
Time series



T-EOFs



RCs



- SSA is good at isolating oscillatory behavior via paired eigenelements.
- SSA tends to lump signals that are longer-term than the window into
  - one or two trend components.

Selected References:

Vautard & Ghil (1989, *Physica* D);
Ghil *et al*. (2002, *Rev. Geophys.)*

SSA-MTM Toolkit, I –
A Free Toolkit for Spectral Analysis

- Developed at UCLA, with collaborations on 3 continents, since 1994.
- Used extensively at the ENS and in various summer schools for teaching spectral methods to various audiences.
- GUI based, for Linux, Unix and Mac OS X platforms.
- Command-line version available.
- Latest developments by A. Groth and D. Kondrashov (UCLA).
- Total cites for top 4 papers ≅ 2000 (WoS, July 2015)
- Hundreds of downloads at every new version.
- Available at: https://dept.atmos.ucla.edu/tcd/ssa-mtm-toolkit .

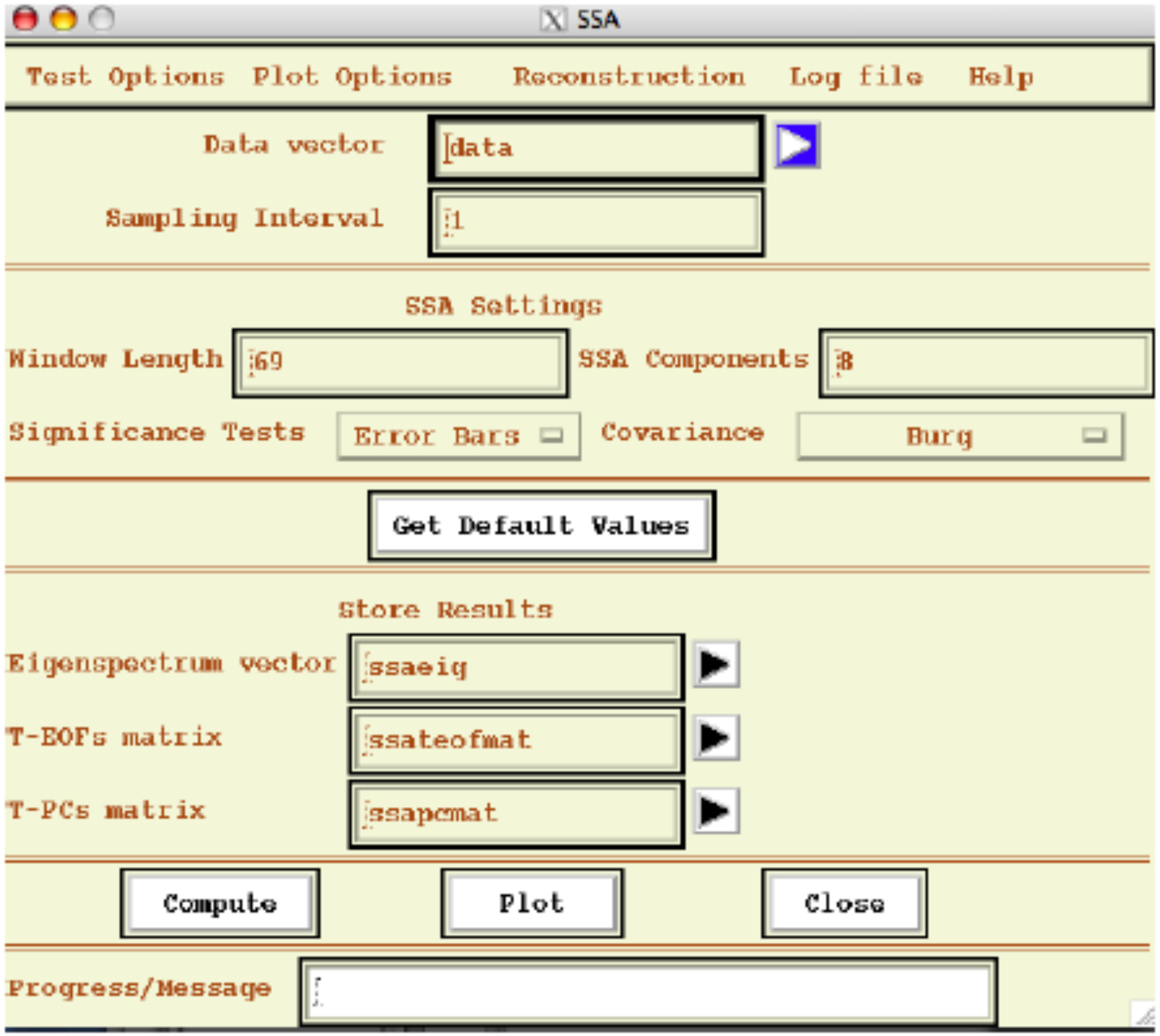
SSA-MTM Toolkit, II – General Goals

- Reduce the variance of the spectral estimate of a time series, based on the periodogram (MTM), correlogram (BT) or other spectral analysis method (e.g., SSA).
- Estimate peak frequencies to “fingerprint” limit cycles of the underlying dynamical system.
- Provide statistical significance tests when such behavior is blurred by “noise.”
- Allow rapid, visual and numerical comparison between the results of different methods: BT, SSA, MEM, MTM.

BT – Blackman-Tukey version of FT (Fourier transform),
MEM – Maximum Entropy Method, MTM – Multi-Taper Method

SSA-MTM Toolkit, III – Targeted audiences

- Non-specialists in time series analysis
  - Reasonable default options
  - Reads ASCII files
- Non-specialists in computer management
  - Precompiled binaries
  - User-friendly interface
- Non-specialists in dynamical systems
  - Better if you do know.
  - No problem if you don't.



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SSA for Southern Oscillation Index (SOI)

SOI = mean monthly values of Δp_s (Tahiti – Darwin)

Results ("undigested") from 1933–1988 time interval (*)

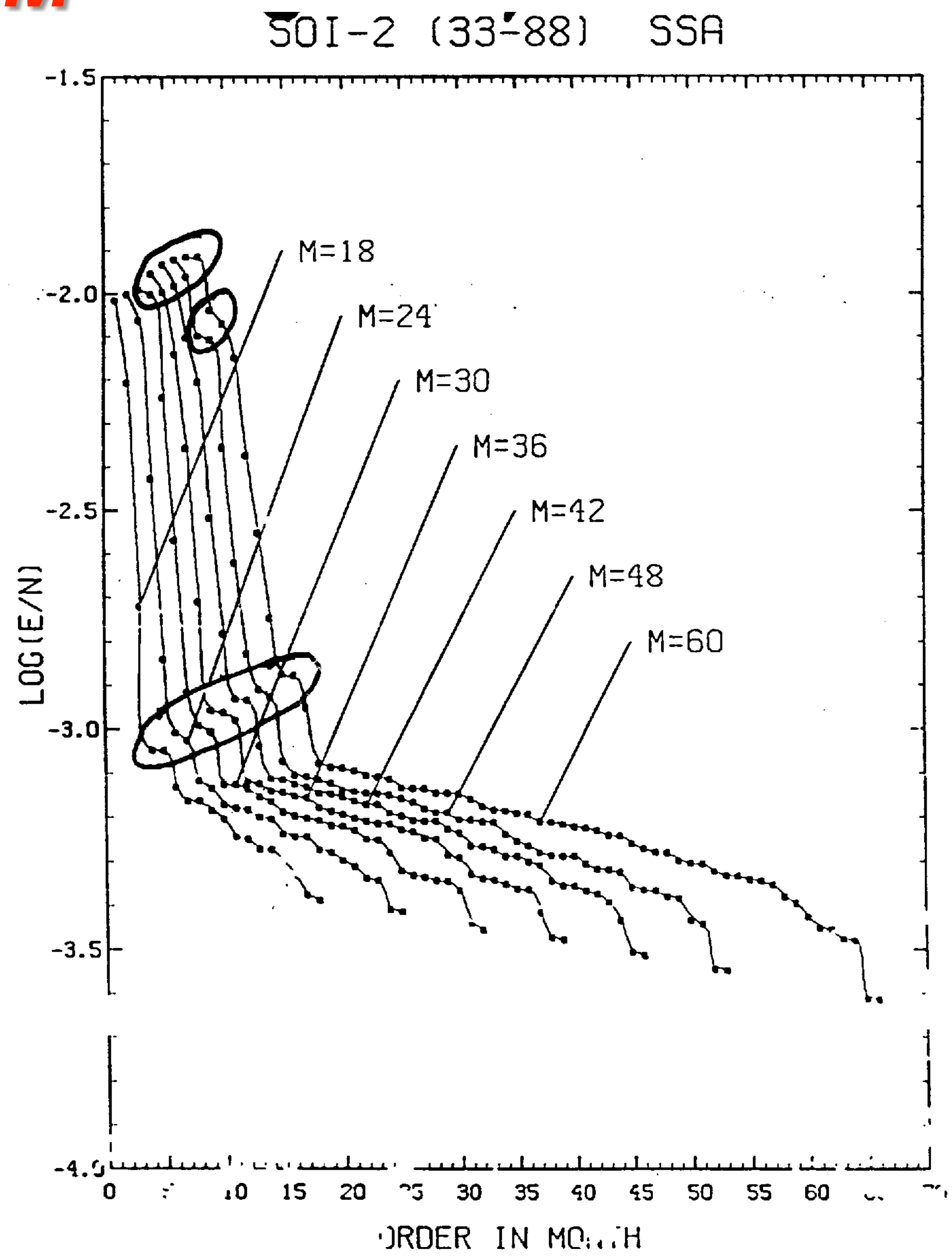
1. For $18 < M < 60$ months, singular spectra show a clear break at $5 < S < 17$ (= "deterministic" part; $M - S$ = "noise");
2. 3 pairs of EOFs stand out:
   EOFs 1 + 2 (27%), 3 + 4 (19.7%), and 9 + 10 (3%);
3. the associated periods are ~
   60 mos. ("ENSO"), 30 mos. (QBO"), and 5.5 mos. (?!)

(*) E. M. ("Gene") Rasmusson, X. Wang, and C.F. Ropelewski, 1990: The biennial component of ENSO variability. *J. Marine Syst.*, **1**, 71–96.

Variable window size *M*

Sampling interval – $\tau_s = 1$ month

Window width $M\tau_s$:
$18\tau_s < \tau_w < 60\tau_s$ or
1.5 yr $< \tau_w < 5$ yr.



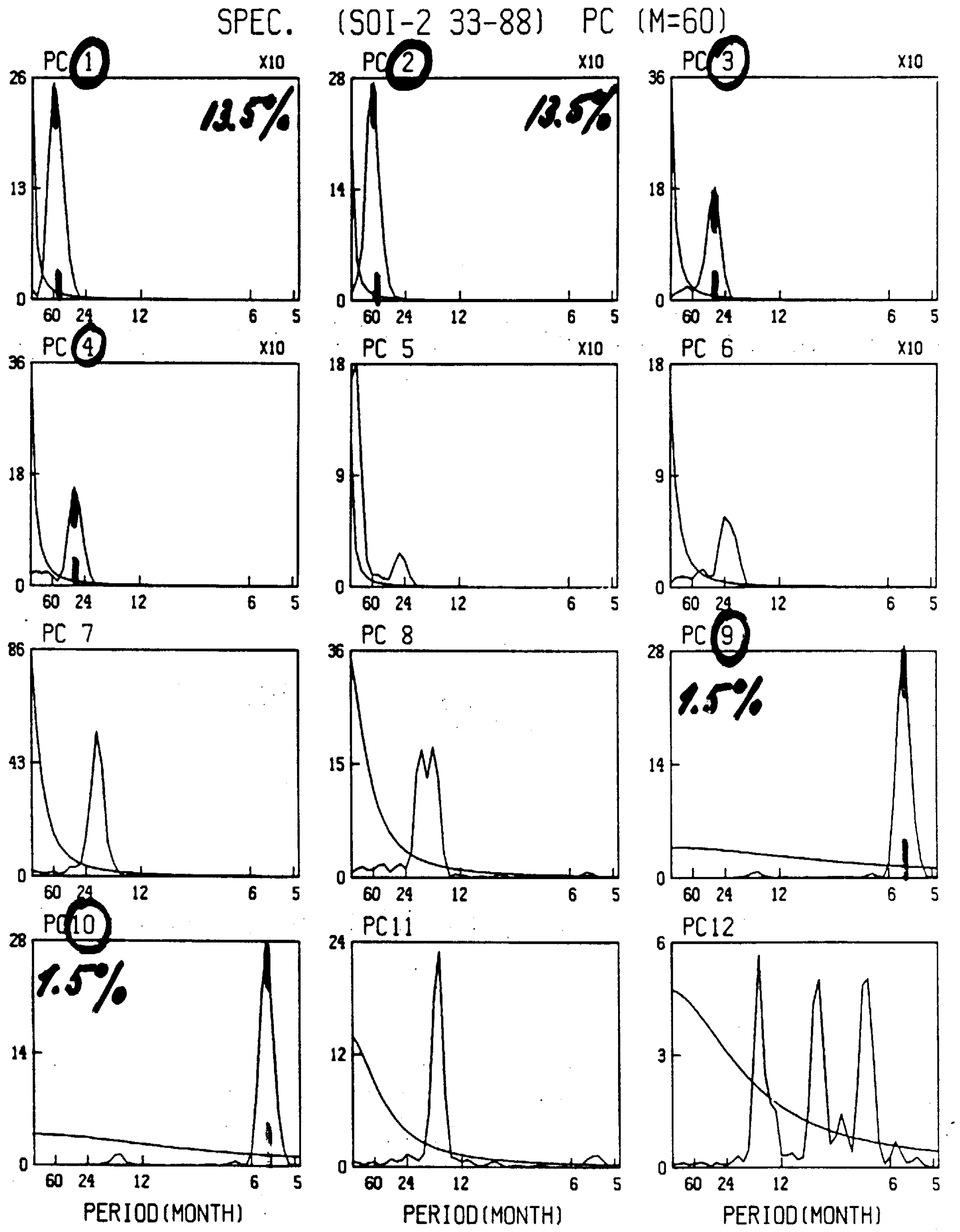
Spectral peaks
(M = 60)

Each principal component (PC) is Fourier analyzed separately; individual variance indicated as well.

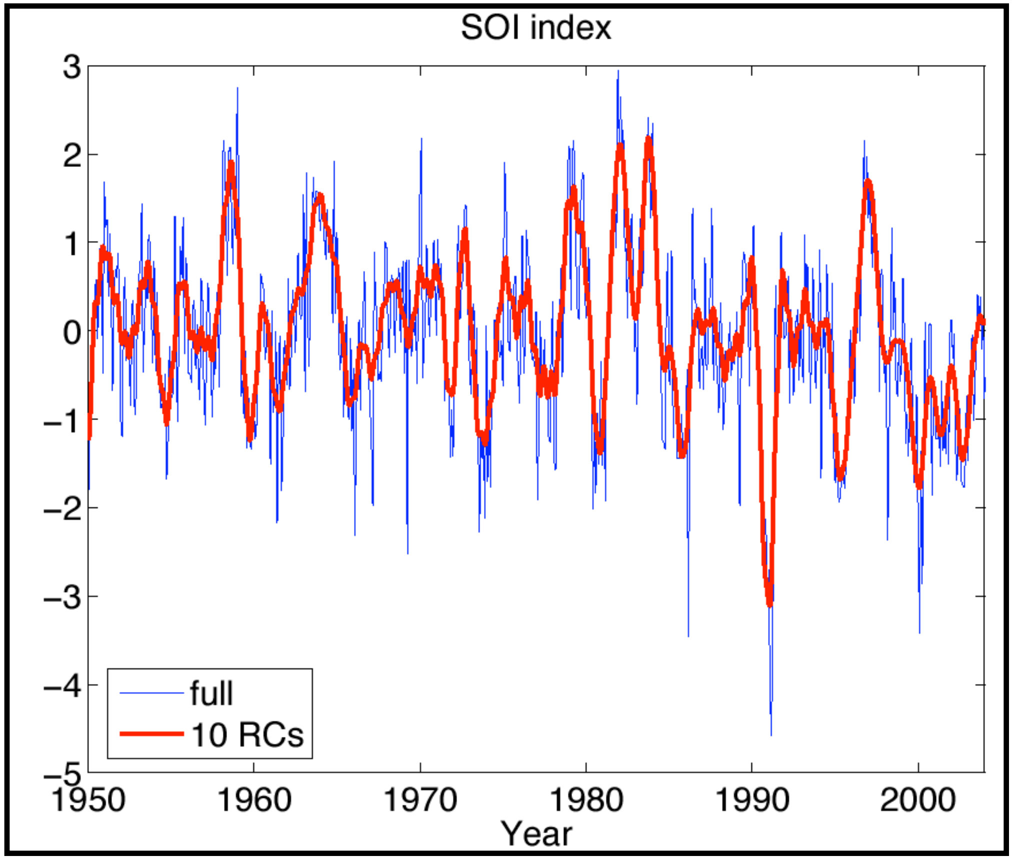
PCs (1+2) – period = 60 months, low-frequency or "ENSO" or quasi-quadrennial (QQ) component;

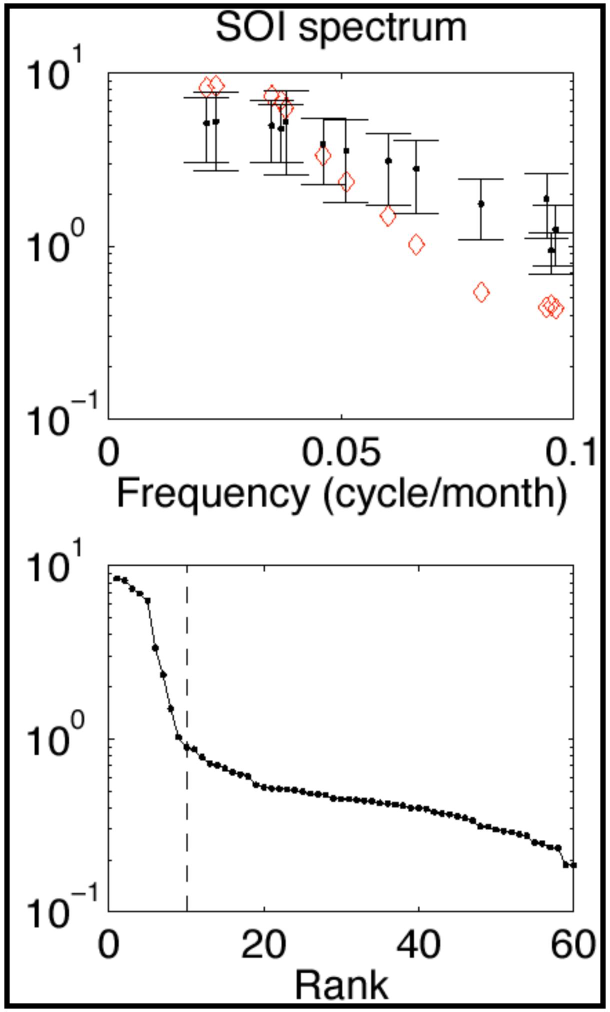
PCs (3+4) – period = 30 months quasi-biennial (QB) component;

PCs (9+10) – period = 5.5 months



Singular Spectrum Analysis (SSA) and M-SSA (cont'd)

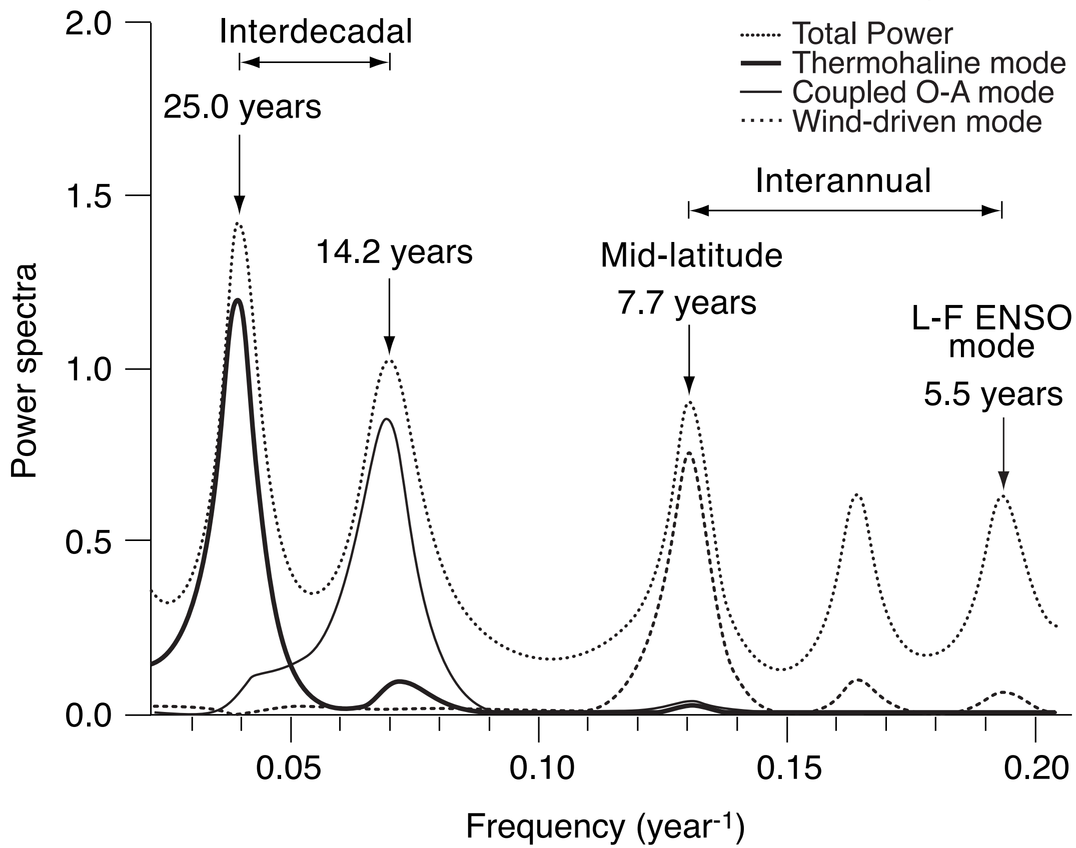




- Break in slope of SSA spectrum distinguishes "***significant***" from "***noise***" EOFs
- Formal Monte-Carlo test (Allen and Smith, 1994) identifies 4-yr and 2-yr ENSO oscillatory modes. A window size of M = 60 is enough to "resolve" these modes in a monthly SOI time series

SSA (prefilter) + (low-order) MEM

- "Stack" spectrum



In good agreement with MTM peaks of **Ghil & Vautard (1991, *Nature*)** for the Jones *et al.* (1986) temperatures & stack spectra of Vautard *et al.* (1992, *Physica D*) for the IPCC "consensus" record (both global), to wit 26.3, 14.5, 9.6, 7.5 and 5.2 years.

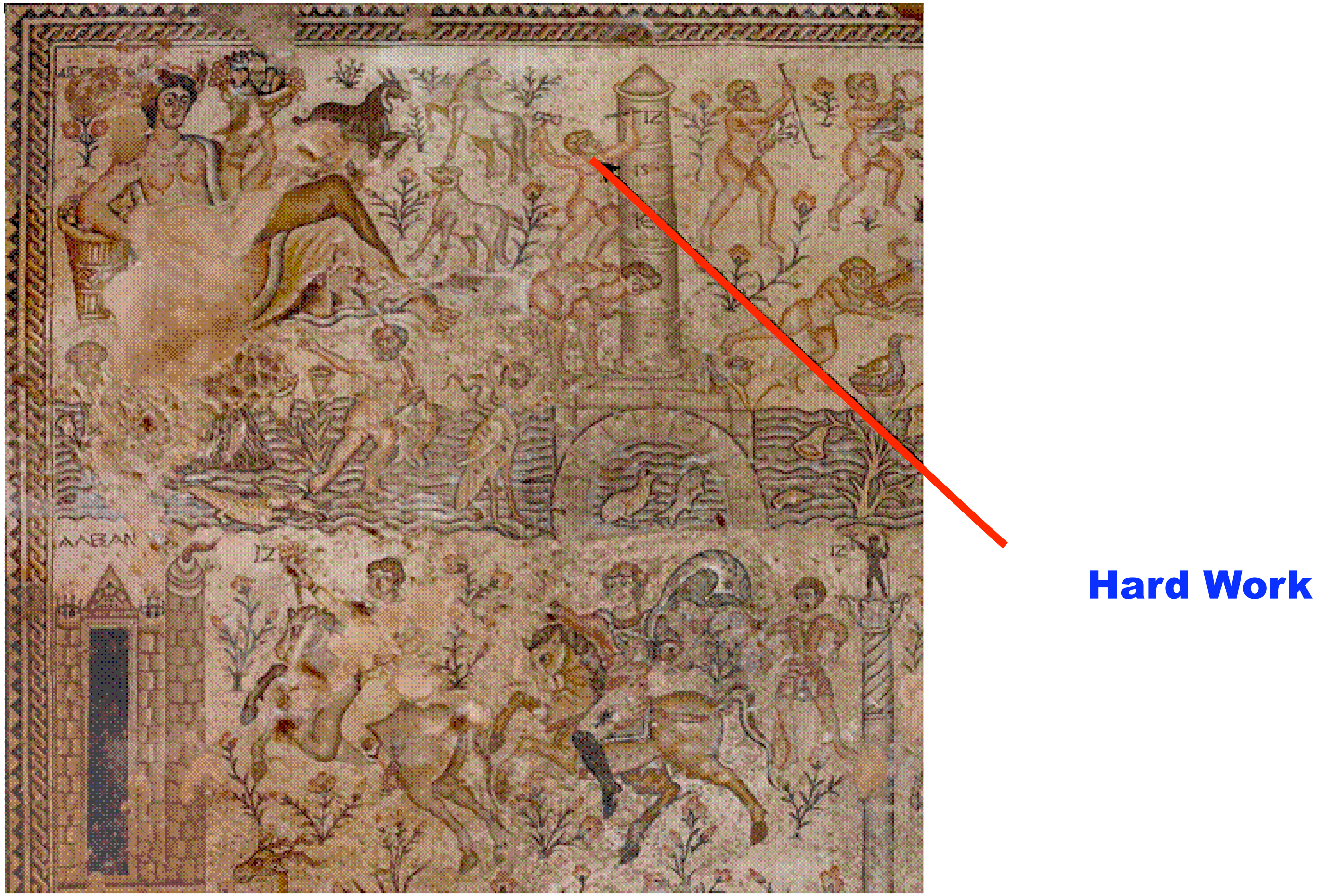
Peaks at 27 & 14 years also in Koch sea-ice index off Iceland (Stocker & Mysak, 1992), etc.
Plaut, Ghil & Vautard (1995, *Science*)

The Nile River Records Revisited:

How good were Joseph's predictions?

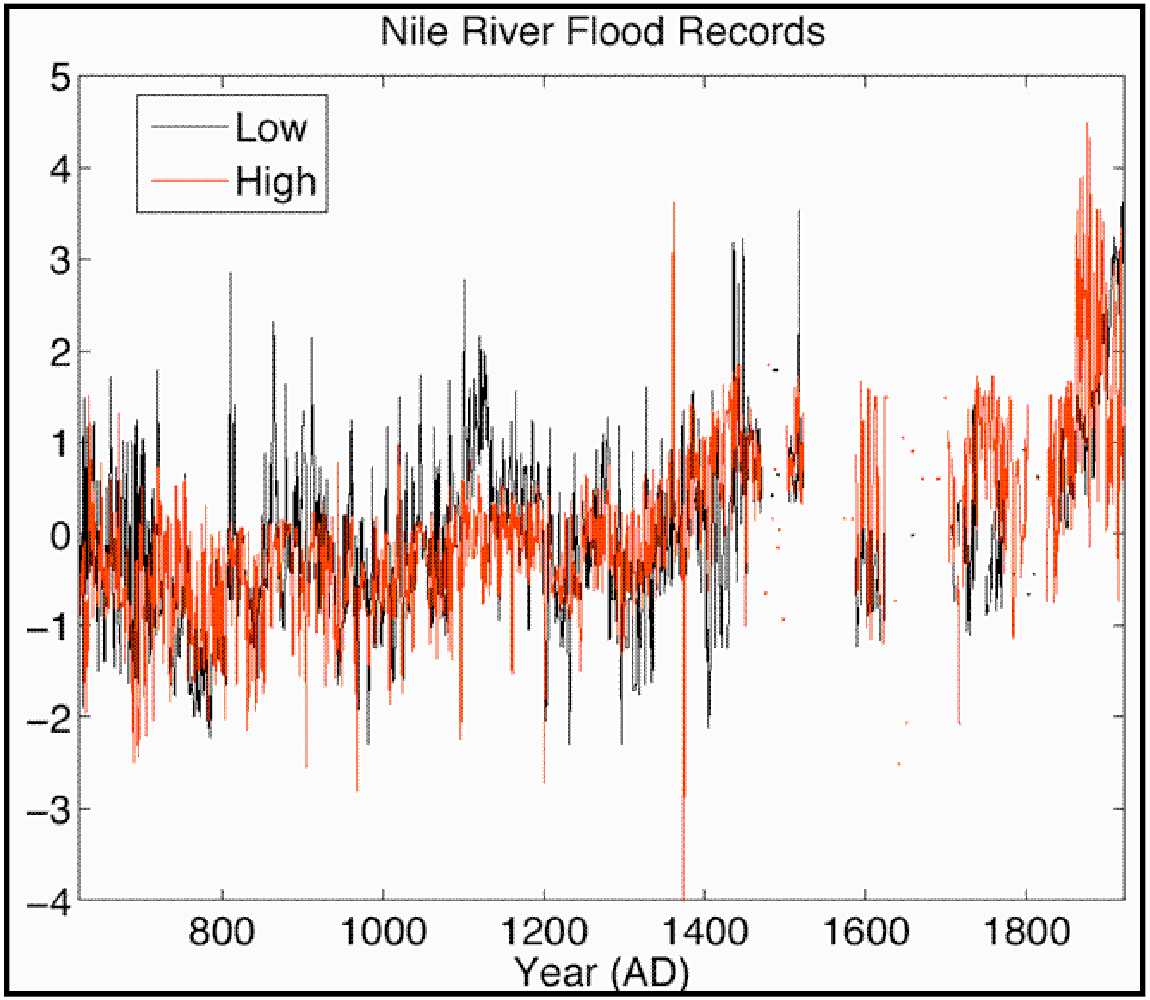
Michael Ghil, ENS & UCLA
Yizhak Feliks, IIBR & UCLA,
Dmitri Kondrashov, UCLA

Why are there data missing?



- Byzantine-period mosaic from Zippori, the capital of Galilee (1st century B.C. to 4th century A.D.); photo by Yigal Feliks, with permission from the Israel Nature and Parks Protection Authority)

Historical records are full of “gaps”....



Annual maxima and minima of the water level at the nilometer on Rodah Island, Cairo.

SSA (M-SSA) Gap Filling

Main idea: utilize both spatial and temporal correlations to iteratively compute self-consistent lag-covariance matrix; M-SSA with $M = 1$ is the same as the EOF reconstruction method of Beckers & Rixen (2003)

Goal: keep "signal" and truncate "noise" — usually a few leading EOFs correspond to the dominant oscillatory modes, while the rest is noise.

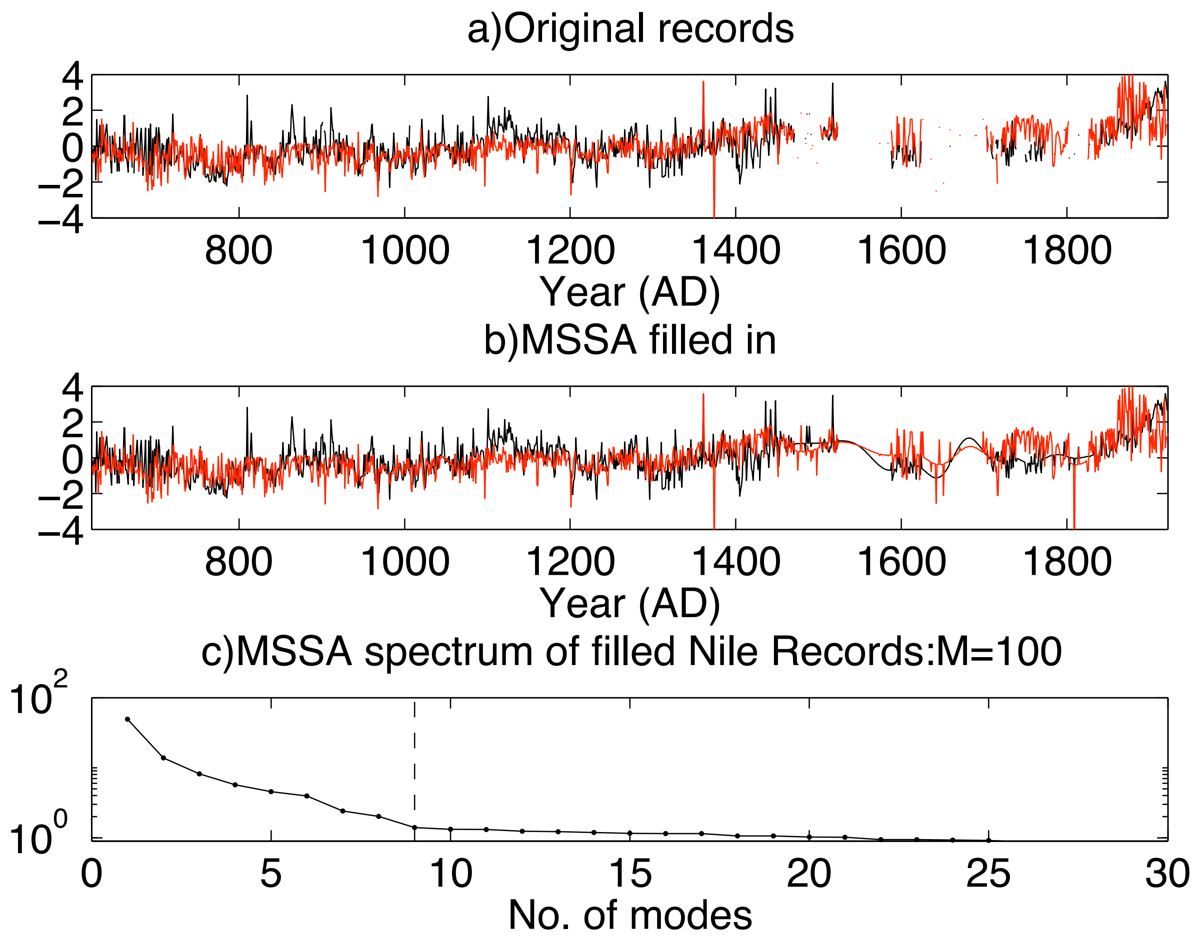
(1) for a given window width M: center the original data by computing the unbiased value of the mean and set the missing-data values to zero.

(2) start iteration with the first EOF, and replace the missing points with the reconstructed component (RC) of that EOF; repeat the SSA algorithm on the new time series, until convergence is achieved.

(3) repeat steps (1) and (2) with two leading EOFs, and so on.

(4) apply cross-validation to optimize the value of M and the number of dominant SSA (M-SSA) modes K to fill the gaps: a portion of available data (selected at random) is flagged as missing and the RMS error in the reconstruction is computed.

Nile River Records



- High level
- Low level

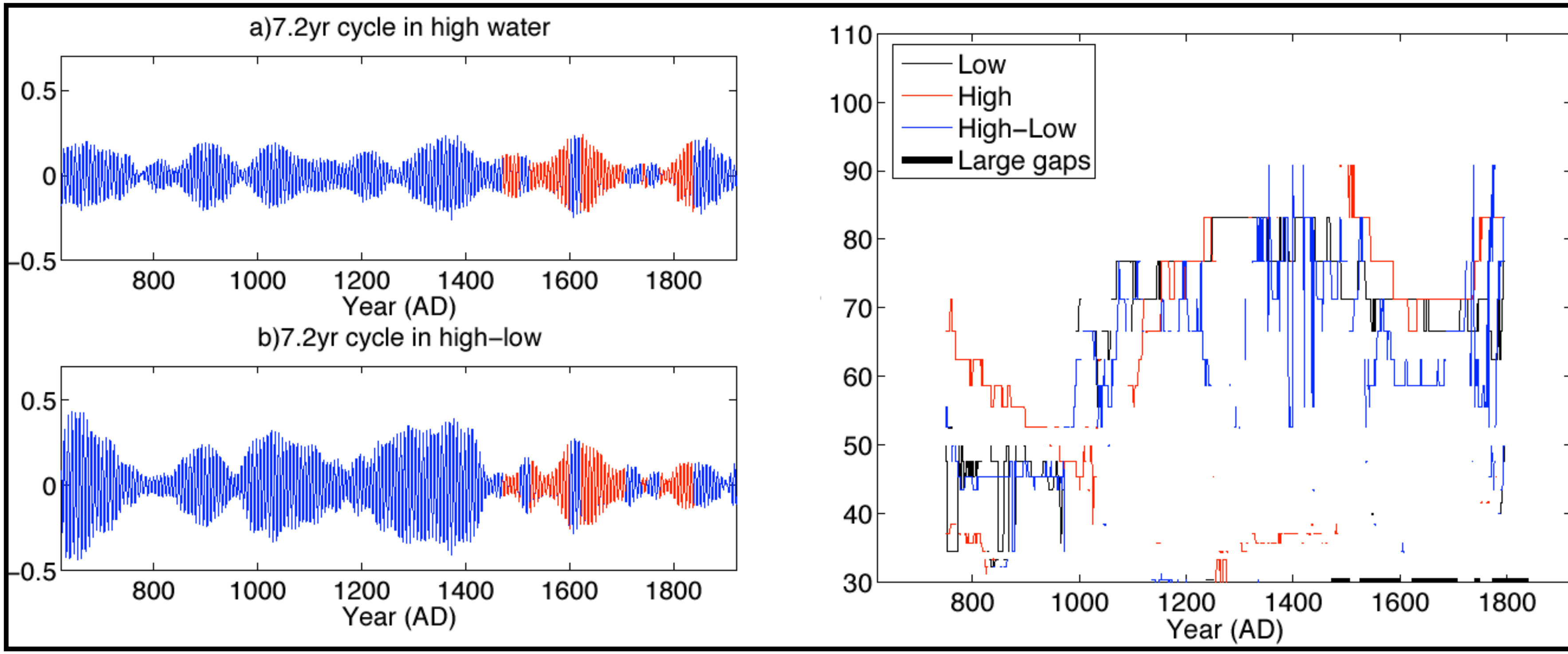
Table 1a: Significant oscillatory modes in short records (A.D. 622–1470)

Periods	Low	High	High-Low
40–100yr	**64** (9.3%)	**64** (6.9%)	**64** (6.6%)
20–40yr		[32]	
10–20yr	**12.2** (5.1%), **18.0** (6.7%)		**12.2** (4.7%), **18.3** (5.0%)
5–10yr	**6.2** (4.3%)	**7.2** **(4.4%)**	**7.3** **(4.4%)**
0–5yr	**3.0** (2.9%), **2.2** (2.3%)	**3.6** (3.6%), **2.9** (3.4%), **2.3** (3.1%)	**2.9** (4.2%),

Table 1b: Significant oscillatory modes in extended records (A.D. 622–1922)

Periods	Low	High	High-Low
40–100yr	**64** (13%)	**85** (8.6%)	**64** (8.2%)
20–40yr		**23.2** (4.3%)	
10–20yr	[12], **19.7** (5.9%)		**12.2** (4.3%), **18.3** (4.2%)
5–10yr	[6.2]	**7.3** (4.0%)	**7.3** (4.1%)
0–5yr	**3.0** (4%), **2.2** (3.3%)	**4.2** (3.3%), **2.9** (3.3%), **2.2** (2.9%)	[4.2], **2.9** (3.6%), **2.2** (2.6%)

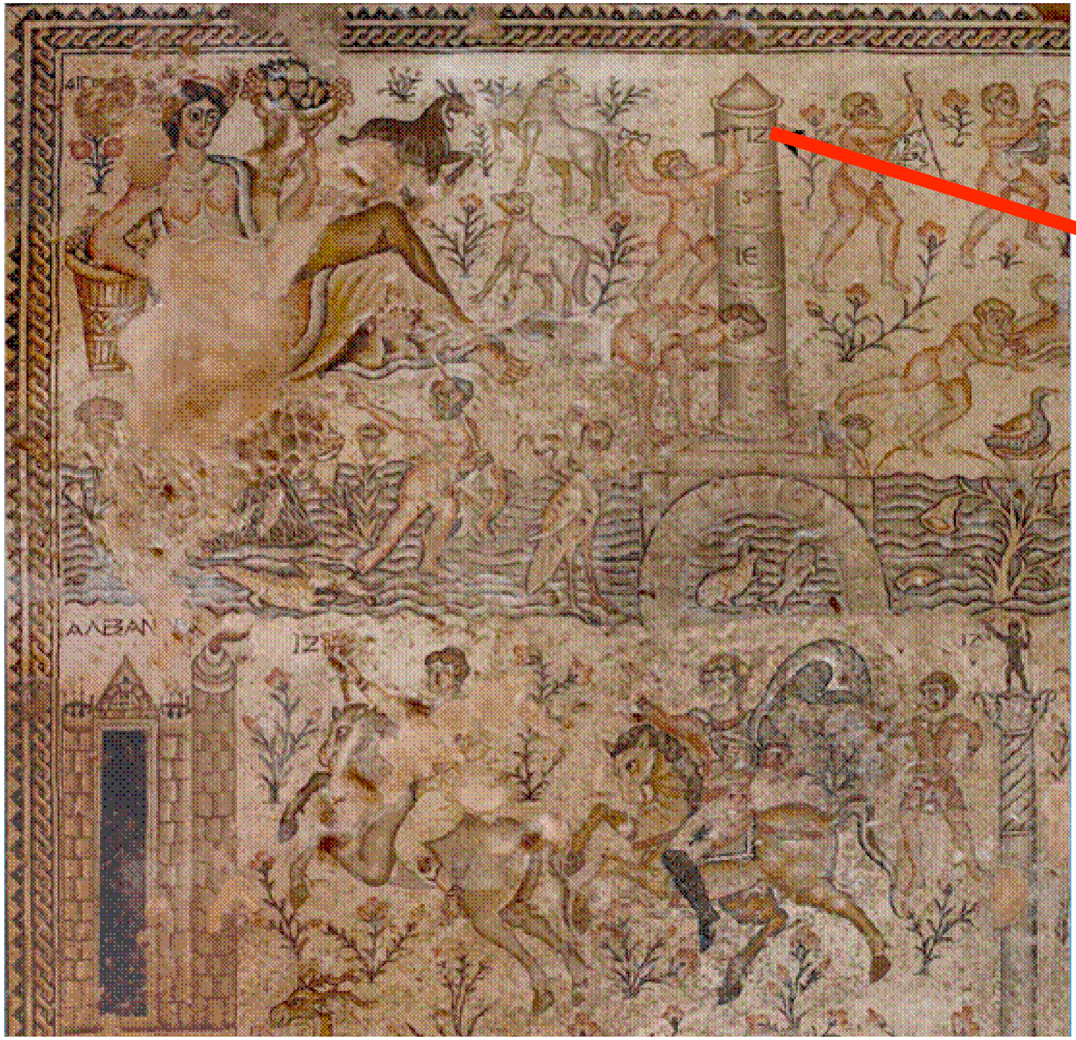
Significant Oscillatory Modes



SSA reconstruction of the 7.2-yr mode in the extended Nile River records: (a) high-water, and (b) difference. Normalized amplitude; reconstruction in the large gaps in red.

Instantaneous frequencies of the oscillatory pairs in the low-frequency range (40–100 yr). The plots are based on multi-scale SSA [Yiou *et al*., 2000]; local SSA performed in each window of width $W = 3M$, with $M = 85$ yr.

How good were Joseph's predictions?



Pretty good!

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Concluding Remarks

- M-SSA allows one to extract spatial + temporal features from time series without any *a priori* hypothesis on the physical or economical processes at work or on the data set's stochastic characteristics.
- We illustrated successful applications to both climatic and economic time series, including ENSO, Nile River floods, US and EU macroeconomic indicators.
- Many other successes in the physical and life sciences, including synchronization of chaotic oscillators, solid-earth geosciences, fish population data, and agent-based predator–prey models.
- Further methodological developments: varimax rotation, Procrustes target target rotation.
- These developments allow application to large data sets in many areas, without prior dimension reduction, e.g. to complete temperature fields.
- Availability of freeware toolkit(*) + other teaching and research aids.

(*) *SSA-MTM Toolkit*, https://dept.atmos.ucla.edu/tcd/ssa-mtm-toolkit .

Significance tests ("garde-fous") in SSA

To check a spectral feature, e.g., an oscillatory pair:

1. Find pair for given data set $\{X_n: n = 1,2, \ldots N\}$ and window width M.
2. Apply statistical significance tests (MC-SSA, etc.).
3. Check robustness of pair by changing M, sampling interval τ_s, etc.
3. Apply additional methods (MTM, wavelets, etc.) and their tests to $\{X_n\}$.
4. Obtain additional time series pertinent to the same phenomenon $\{Y_m\}$, etc.
5. Apply steps (1)–(3) to these data sets.
6. Use multi-channel SSA (M-SSA) and other multivariate methods to check mutual dependence between $\{X_n\}$, $\{Y_m\}$, etc.
7. Based on steps (1)–(6), try to provide a physical explanation of the mode.
8. Use (7) to predict an as-yet-unobserved feature of the data sets.
9. If this new feature is found in new data, go on to next problem.
10. If not, go back to an earlier step of this list.

(*) **Ghil, M.**, M. R. Allen, M. D. Dettinger, K. Ide, D. Kondrashov, M. E. Mann, A. W. Robertson, A. Saunders, Y. Tian, F. Varadi, and P. Yiou, 2002: Advanced spectral methods for climatic time series, *Rev. Geophys.*, **40**(1), pp. **3**.1–**3**.41, doi: 10.1029/2000RG000092.

Some general references

Classical, paper-based

Blackman, R. B., & J. W. Tukey, 1958: *The Measurement of Power Spectra*, Dover, Mineola, NY.

Broomhead, D. S., King, G. P., 1986a. Extracting qualitative dynamics from experimental data. *Physica D*, **20**, 217–236.

Chatfield, C., 1984: *The Analysis of Time Series: An Introduction*, Chapman & Hall, New York.

Ghil, M., *et al*., 2002: Advanced spectral methods for climatic time series, *Rev. Geophys.,* **40**, doi:10.1029/2001RG000092.

Hannan, E. J., 1960: *Time Series Analysis*, Methuen, London/Barnes & Noble, New York, NY, 152 pp.

Loève, M., 1978*: Probability Theory*, *Vol. II, 4th ed.,* Graduate Texts in Mathematics, vol. 46, Springer-Verlag, ISBN 0-387-90262-7.

Percival, D. B.,& A. T. Walden, *Spectral Analysis for Physical Applications*, Cambridge Univ. Press, 1993.

Web-based

https://dept.atmos.ucla.edu/tcd/ssa-mtm-toolkit.

Reserve slides

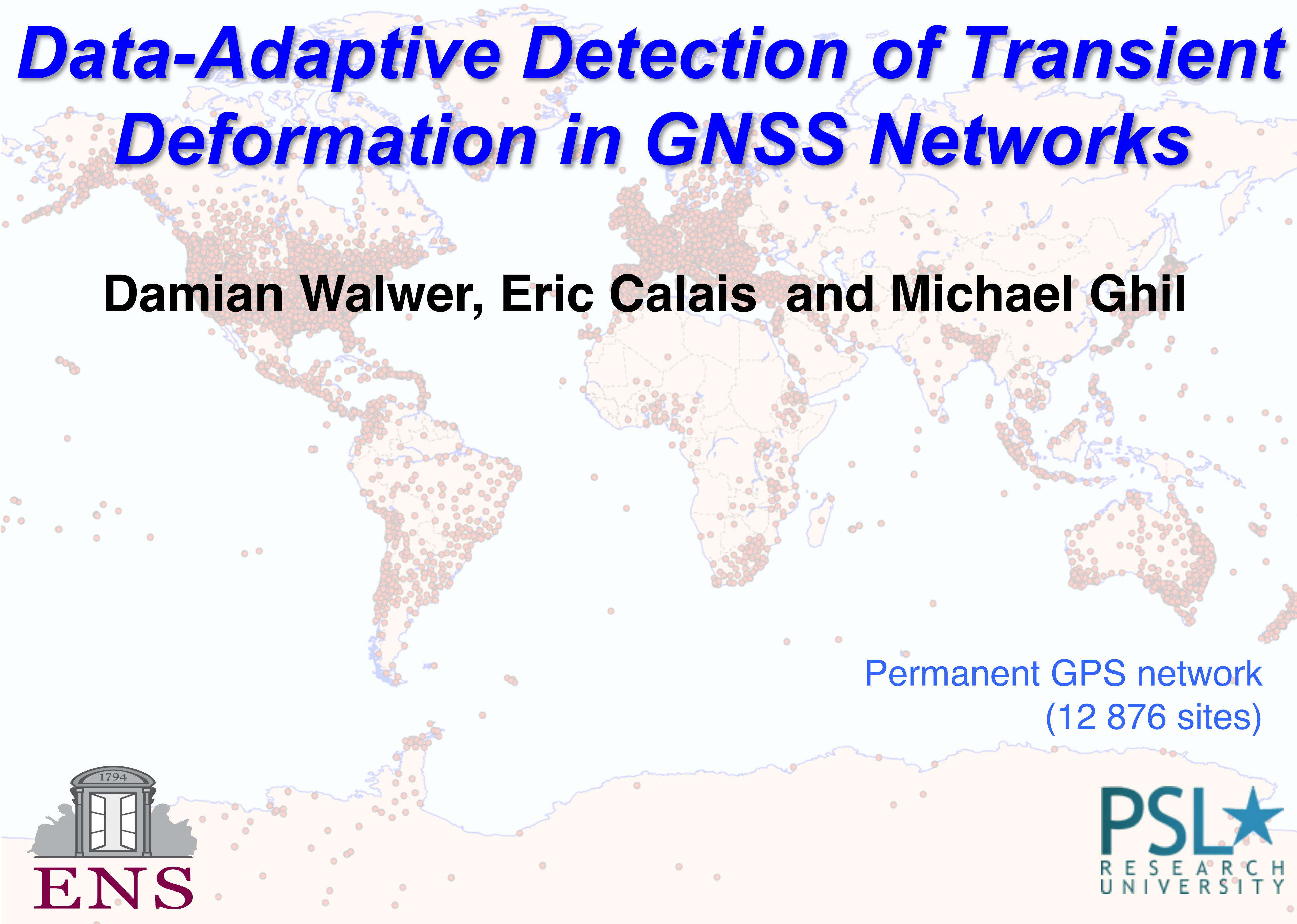
Diapos de réserve

Outline

- Time series analysis
  - The "smooth" and "rough" part of a time series
  - Oscillations and nonlinear dynamics
- Singular spectral analysis (SSA)
  - Principal components in time and space
  - The SSA-MTM Toolkit
- The Nile River floods
  - Longest climate-related, instrumental time series
  - Gap filling in time series
  - NAO and SOI impacts on the Nile River
- Concluding remarks
  - Cautionary remarks ("garde-fous")
  - References

Data-Adaptive Detection of Transient Deformation in GNSS Networks

Damian Walwer, Eric Calais and Michael Ghil



ENS

PSL RESEARCH UNIVERSITY

Motivation & Outline

1. Data sets in the geosciences are often short and contain errors: this is both an obstacle and an incentive.

2. Phenomena in the geosciences often have both regular ("cycles") and irregular ("noise") aspects.

3. Different spatial and temporal scales:
   one person's noise is another person's signal.

4. Need both deterministic and stochastic modeling.

5. Regularities include (quasi-)periodicity ➔ spectral analysis via "classical" methods (see SSA-MTM Toolkit).

6. Irregularities include scaling and (multi-)fractality ➔ "spectral analysis" via Hurst exponents, dimensions, etc. (see Multi-Trend Analysis, MTA)

7. Does some combination of the two, + deterministic and stochastic modeling, provide a pathway to prediction?

For details and publications, please visit these two Web sites:

TCD http://www.atmos.ucla.edu/tcd/ ➔ key person – Dmitri Kondrashov!

E2-C2 http://www.ipsl.jussieu.fr/~ypsce/py_E2C2.html

Time Series in Nonlinear Dynamics

The 1980s — decade of greed & fast results

(LBOs, junk bonds, fractal dimension).

Packard *et al*. (1980), Roux *et al*. (1980);

Mañe (1981), Ruelle (1981), Takens (1981);

- Method of delays: $\ddot{x}_i = f_i(x_1, ..., x_n) \Leftrightarrow x^{(n)} = F(x^{(n-1)}, ..., x)$

$$\ddot{x} = F(x, \dot{x}) \Rightarrow \begin{cases} \dot{x} = y, \\ \dot{y} = F(x, y) \end{cases}$$

Differentiation ill-posed ⇒ use differences instead!

1st Problem — smoothness:

Whitney embedding lemma doesn't apply to most attractors (e.g.,Lorenz)

2nd Problem — noise;

3rd Problem — sampling: long recurrence times.

- Some rigorous results on convergence:

Smith (1988, *Phys. Lett. A*), Hunt (1990, *SIAM J. Appl. Math.*)

Power Law for Spectrum

$S(f) \sim f^{-p}$ + poles

i.e. linear in log-log coordinates

For a 1st-order Markov process or “red noise” $p = 2$

“Pink” noise, $p = 1$ ($1/f$, flicker noise)

“White” noise, $p = 0$

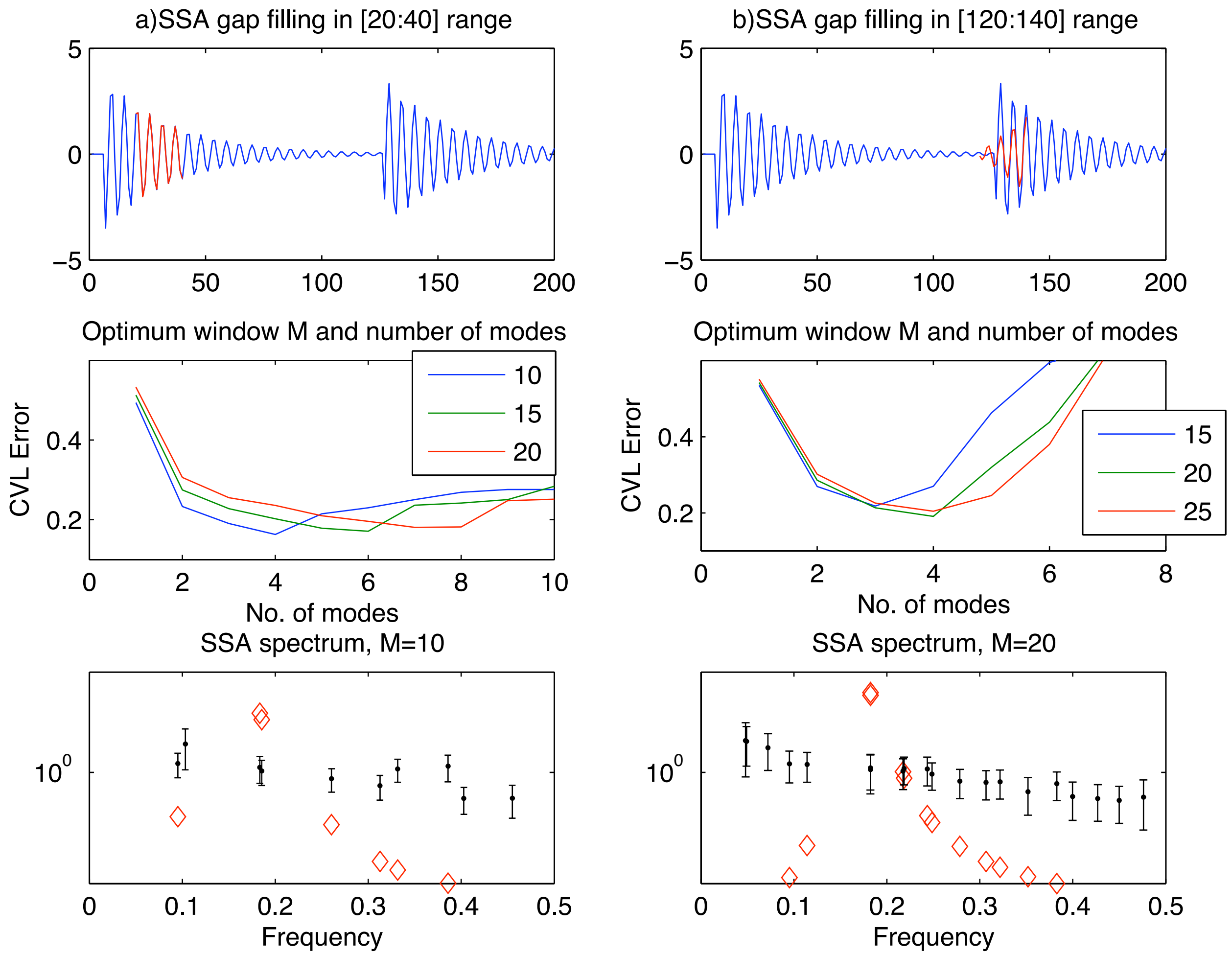
Low-order dynamical (deterministic) systems

have exponential decay of $S(f)$ (linear in log-linear coordinates)

e.g. for Smale horseshoe $\forall k \quad \exists 2^k$ unstable orbits of period k

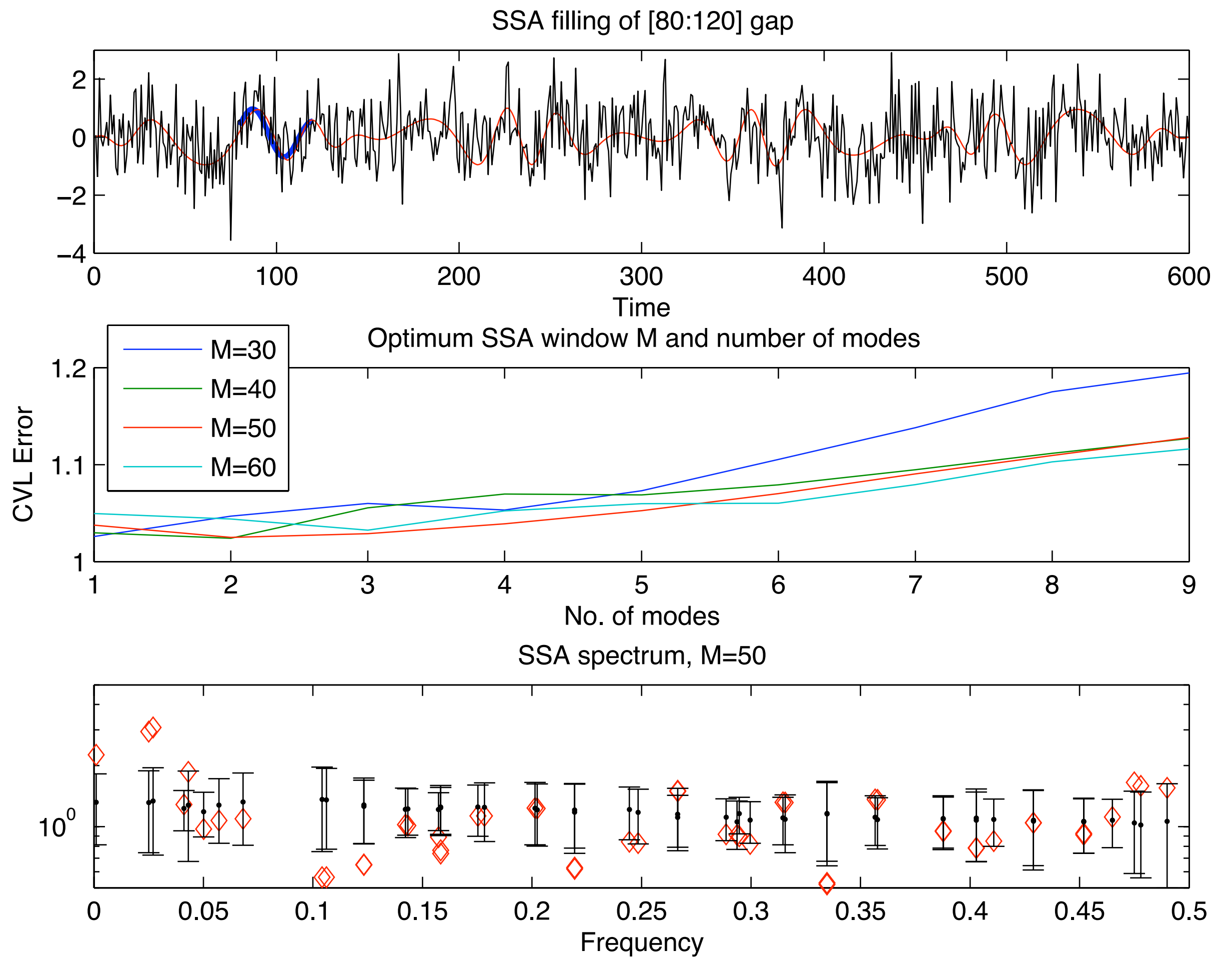
N.B. Bhattacharaya, Ghil & Vulis (1982, *J. Atmos. Sci*.) showed a spectrum $S \sim f^{-2}$ for a nonlinear PDE with delay (doubly infinite-dimensional)

Synthetic I: Gaps in Oscillatory Signal



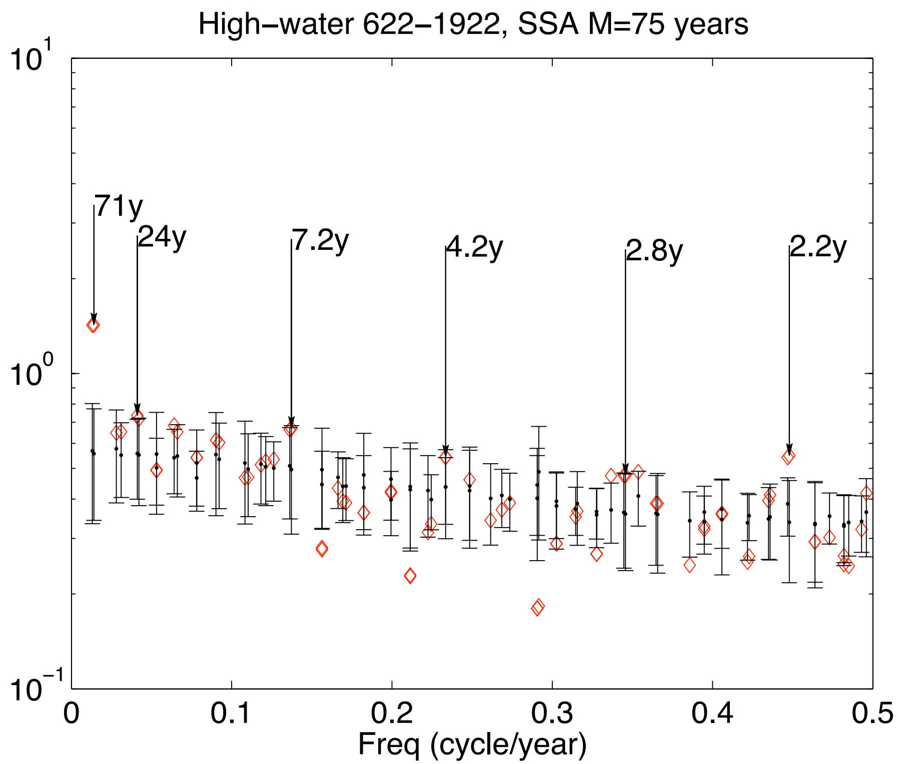
- Very good gap filling for smooth modulation; OK for sudden modulation.

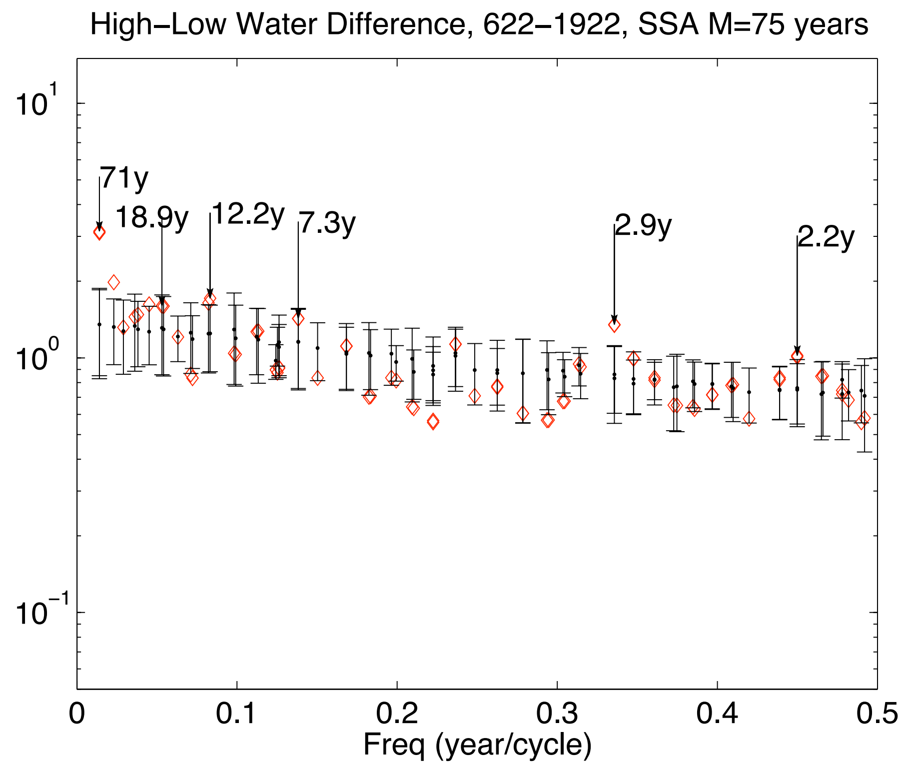
Synthetic II: Gaps in Oscillatory Signal + Noise



$$x(t) = sin(\frac{2\pi}{300}t) * cos(\frac{2\pi}{40}t + \frac{\pi}{2}sin\frac{2\pi}{120}t)$$

MC-SSA of Filled-in Records





SSA results for the extended Nile River records;

arrows mark highly significant peaks (at 95%), in both SSA and MTM.

Some references

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Advanced Spectral Methods, Nonlinear Dynamics, and the Nile River

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Motivation

1. Climatic time series have typically broad peaks on top of a continuous, "warm-colored" background ➔ ***Method***

2. Connections to nonlinear dynamics ➔ ***Theory***

3. Need for stringent statistical significance tests ➔ ***Toolkit***

4. Applications to analysis and prediction ➔ ***Examples***

Joint work with: M. R. Allen, M. D. Dettinger, K. Ide, N. Jiang, C. L. Keppene, D. Kondrashov, M. Kimoto, M. E. Mann, J. D. Neelin, M. C. Penland, G. Plaut, A. W. Robertson, A. Saunders, D. Sornette, S. Speich, C. M. Strong, C. Taricco, Y.-d. Tian, Y. S. Unal, R. Vautard, & P. Yiou (on 3 continents).

http://www.atmos.ucla.edu/tcd

Type of noise used in the toolkit

- Red noise:
  - AR(1) random process: $X(t+1) = aX(t) + b(t)$
  - Decreasing spectrum (due to inertia)

$$\boxed{C_X(\tau) = \frac{\sigma^2 a^{|\tau|}}{1 - a^2}}$$

$$P_X(f) = C_X(0) \frac{1 - a^2}{1 - 2a\cos 2\pi f + a^2}$$

Monte Carlo SSA

(Allen et Smith, *J. Clim.*, 1995)

Goal: Assess whether the SSA spectrum can reject the null hypothesis that the time series is red noise.

Procedure:

- Estimate red noise parameters with same variance and auto-covariance as the observed time series $X(t)$
- Compare the pdf of the projection of the noise covariance onto the data EOFs:

$$\Lambda_B = R_X^t \underbrace{C_R}_{\text{Covar. bruit rouge}} \overbrace{R_X}^{\text{EOFs données}}$$

The null hypothesis is rejected using the pdf of Λ_B.

Monte Carlo SSA: red noise test